dec test revision

[SQA] 1. (*a*) Express $f(x) = x^2 - 4x + 5$ in the form $f(x) = (x - a)^2 + b$. **2**

(*b*) On the same diagram sketch:

(i) the graph of $y = f(x)$;

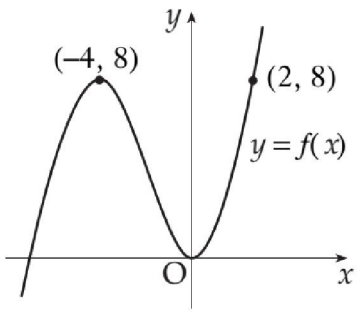
(ii) the graph of $y = 10 - f(x)$. **4**

(*c*) Find the range of values of x for which $10 - f(x)$ is positive. **1**

[SQA] 2. The diagram shows a sketch of the function $y = f(x)$.

(*a*) Copy the diagram and on it sketch the graph of $y = f(2x)$. **2**

(*b*) On a separate diagram sketch the graph of $y = 1 - f(2x)$. **3**



[SQA] 3. Differentiate $\sin 2x + \frac{2}{\sqrt{x}}$ with respect to x. **4**

[SQA] 4. Find the coordinates of the point on the curve $y = 2x^2 - 7x + 10$ where the tangent to the curve makes an angle of $45°$ with the positive direction of the x-axis. **4**

[SQA] 5. Find the x-coordinate of each of the points on the curve $y = 2x^3 - 3x^2 - 12x + 20$ at which the tangent is parallel to the x-axis. **4**

[SQA] 6. (*a*) $f(x) = 2x + 1$, $g(x) = x^2 + k$, where k is a constant.

(i) Find $g(f(x))$. **(2)**

(ii) Find $f(g(x))$. **(2)**

(*b*) (i) Show that the equation $g(f(x)) - f(g(x)) = 0$ simplifies to $2x^2 + 4x - k = 0$. **(2)**

(ii) Determine the nature of the roots of this equation when $k = 6$. **(2)**

(iii) Find the value of k for which $2x^2 + 4x - k = 0$ has equal roots. **(3)**

[SQA] 7. Show that the equation $(1-2k)x^2 - 5kx - 2k = 0$ has real roots for all integer values of k. **5**

[SQA] 8. When $f(x) = 2x^4 - x^3 + px^2 + qx + 12$ is divided by $(x-2)$, the remainder is 114.

One factor of $f(x)$ is $(x+1)$.

Find the values of p and q. **5**

[SQA] 9. Given that $\tan\alpha = \frac{\sqrt{11}}{3}$, $0 < \alpha < \frac{\pi}{2}$, find the exact value of $\sin 2\alpha$. **3**

[SQA] 10.

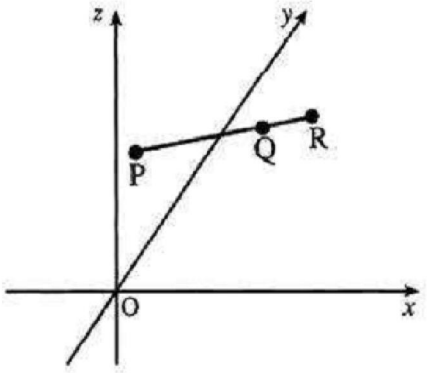
(*a*) Show that $2\cos 2x^\circ - \cos^2 x^\circ = 1 - 3\sin^2 x^\circ$. **2**

(*b*) Hence solve the equation $2\cos 2x^\circ - \cos^2 x^\circ = 2\sin x^\circ$ in the interval $0 \leq x < 360$. **4**

11. Solve $2\cos 2x - 5\cos x - 4 = 0$ for $0 \leq x < 2\pi$. **5**

[SQA] 12. A is the point $(2,-5,6)$, B is $(6,-3,4)$ and C is $(12,0,1)$. Show that A, B and C are collinear and determine the ratio in which B divides AC. **4**

[SQA] 13. **Relative to the axes shown and with an appropriate scale, P(–1, 3, 2) and Q(5, 0, 5) represent points on a road. The road is then extended to the point R such that $\vec{PR} = \frac{4}{3}\vec{PQ}$.**



(*a*) **Find the coordinates of R.** **(3)**

(*b*) **Roads from P and R are built to meet at the point S (–2, 2, 5). Calculate the size of angle PSR.** **(7)**